Problem 1. Repeated games

Consider two wireless transmitters, which can transmit at high power (H) or at low power (L). The transmission rate that they can achieve depends on both their transmission power and on the other player's decision, according to the following payoff matrix:

$$\begin{array}{c c} & \begin{array}{cc} L & H \end{array} \\ \begin{array}{c} L \\ H \end{array} & \begin{bmatrix} (2,2) & (0,3) \\ (3,0) & (1,1) \end{bmatrix} \end{array}$$

They both want to maximize their transmission rate, and they have to transmit T packets. They can decide the transmission power for each packet, and they have to make their decision simultaneously, without knowing what the other transmitter will do. Therefore, the action space for each player is

$$\Gamma_i = \underbrace{\{L, H\} \times ... \times \{L, H\}}_{T \text{ times}}$$

and each action is a sequence of T symbols such as $\gamma_i = \underbrace{(L, H, ..., H)}_{T \text{ symbols}}$.

The payoff for an action γ_i is the sum of the payoffs for transmission of each packet.

a) How many actions does each player have?

b) Draw the matrix representation of the game for $T = 2$ packets.

c) Find a Nash Equilibrium for the game with $T = 2$ packets

d) What is the Nash Equilibrium for $T = 200$?

e) Now assume each player can revise its action after seeing the outcome of the game at the previous stage. Consider the trigger strategy: a player i playing the trigger strategy starts by playing L, and keeps playing L. However, if the other player plays H, then player i changes to H in the next stage, and keeps playing H for the rest of the game. What would be the outcome of the game under the trigger strategy? Is this a Nash equilibrium strategy? Justify your answer.

In contrast to the case above, now they can observe the other player's strategy after they both transmit a packet (after each stage of the game).

f) How many strategies does each player have?

g) Draw the game in extensive form for $T = 2$ packets.

h) Determine the subgame perfect equilibrium for $T = 2$.

i) How many information sets does each player have for $T = 200$? Determine the subgame perfect equilibrium in this case.

Consider now the case in which players have to transmit packets indefinitely $T = \infty$. To make the problem well-posed we need the players' payoff to be finite. So, consider the following payoff for each player i:

$$J_i = (1 - \delta) \sum_{t=0}^{T} \delta^t J_i(t)$$

where $1/2 < \delta < 1$ and $J_i(t)$ is the payoff of player i at stage (packet) $t = 1, 2, ...$ (as written in the matrix above).

j) Consider the strategy in which both players always play H. What is the outcome, when $T = \infty$? Is this a Nash Equilibrium strategy?

k) Consider the trigger strategy: a player i playing the trigger strategy starts by playing L, and keeps playing L. However, if the other player plays H, then player i changes to H, and keeps playing H for the rest of the game. What is the outcome, when $T = \infty$, if all players play the trigger strategy? Is this a Nash Equilibrium strategy?

For the next exercise, you would need to use the Hespanha, Noncooperative Game Theory, 2015, available online from EPFL library website (link provided in the course Moodle cite).

Problem 2. Behavioral strategies in zero-sum games

Solve Exercise 8.3 from Hespanha book.